

INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2015-16 Semester II

Large Sample Statistical Methods
Final Examination

Points for each question is in brackets. Total Points 100.

Students are allowed to bring 4 pages (front and back) of hand-written notes

Duration: 3 hours

1. (15) If F is discrete, then show that $F_n \Rightarrow F$ iff for each x in the support of F , $P(X_n = x) \rightarrow P(X = x)$.
2. (10) A sequence of random variables X_n is defined as X_n = the number of trials required to obtain the first success when the probability of success in each trial is $1/n$. Find the asymptotic distribution of X_n/n .
3. (15) Consider the following sequence of $2n$ independent random variables:

$$Y_{i1}, Y_{i2} \sim \mathcal{N}(\mu_i, \sigma^2), \quad i = 1, \dots, n.$$

Find the MLE of σ^2 . Is the MLE consistent?

4. (15) Let $X_n \sim \text{Bin}(n, p)$.
 - (a) Find the asymptotic distribution of $\sqrt{n}(X_n/n - p)$.
 - (b) Find a consistent estimator of $p(1 - p)$.
 - (c) Use the results above and Slutsky's theorem to obtain approximate confidence intervals for p .
5. (15) Show that the Pearson chi-square statistic is a Wald statistic. Hence derive its asymptotic distribution under the null hypothesis.
6. (15) Dvoretzky, Kiefer, and Wolfowitz(1956) showed that when X_1, \dots, X_n are iid uniform(0,1), then there exists a finite positive constant C such that

$$P(D_n > d) \leq C \exp\{-2nd^2\}$$

for all n , where D_n is the Kolmogorov Smirnov statistic. Using the above result show the following:

- (a) The result is true for $X_1, \dots, X_n \stackrel{iid}{\sim} F$ for any distribution F .
 - (b) $D_n \rightarrow 0$ with probability one.
7. (15) Consider the U-statistic $\frac{2}{n(n-1)} \sum_{1 \leq i < j \leq n} I(X_i + X_j > 0)$, where I denotes the indicator function. Find the asymptotic distribution of this statistic when F is symmetric about zero.